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APPLICATION OF THE ROBUST ESTIMATE IN SLR DATA PREPROCESSING

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ABSTRACT

M-estimator, one kind of robust estimator, has been used in SLR data preprocessing. It has been shown that the M-estimator has 50% or more breakdown point.

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INTRODUCTION

There are three purposes in preprocessing from a pass of raw satellite range measurements:

a) Correcting system errors for raw SLR data and forming observational files;

b) Fitting a smoothing function to the range residuals from the predicted orbit, rejecting noises and outliers then estimating accurate for this pass;

c) Forming QL, FL and NP data files.

The second term is very impotant for data preprocess, because the smoothing function will have effect on quality of NP data.

The smoothing function we used is simply a polynomial in time. Generaly, the least squares (LS) estimation is used to solve the parameters of polynominal. But, the LS estimation is not a robust estimetion. Sometime, there are a large number of noises in raw SLR data, especialy those passes are in daylight, the solution of the LS estimation will converge to false values.

In this paper, M-estimator, one kind of robust estimator has been used in SLR data preprocessing. It has been shown that the M-estimators has 50% or more breacdown point ε^*. The breakdown point means that, when the probability of noises ε, increases to ε^*, this method will fail.

M-ESTIMATOR

The linear equation is writen:

$$y_i = X_i^{\tau}\theta + e_i \tag{1}$$

where

y_i are observations

θ is the vector of paramaters to be estimated
X_i is the vector of coefficients
e_i are random errors.

The M-estimator, called Maximum Likelihood Type Estimator, is such an estimator which makes the following objective function minimum:

$$\sum_{i=1}^{N} F\{(y_i - X_i^{\tau}\hat{\theta})/\sigma\} = \min. \tag{2}$$

$\hat{\theta}$ are values estimated for θ
σ^2 is variance
$F\{\cdot\}$ is an even function

Different objective functions have different M-estimator. In this paper we used Hampel estimator, here

$$F(r_i) = \begin{cases} \frac{1}{2} r_i^2 & |r_i| \le \lambda_0 \sigma \\ \lambda_0 \sigma (|r_i| - \frac{1}{2}\lambda_0 \sigma) & \lambda_0 \sigma < |r_i| \le \lambda_1 \sigma \\ \frac{\lambda_0}{\lambda_2 - \lambda_1}(\lambda_2 \sigma |r_i| - \frac{1}{2} r_i^2) & \\ \qquad - \frac{\lambda_1}{\lambda_2 - \lambda_1} \frac{\lambda_0}{2} \sigma^2 - \frac{1}{2}\lambda_0^2 \sigma^2 & \lambda_1 \sigma < |r_i| \le \lambda_2 \sigma \\ \lambda_0 (\lambda_2 + \lambda_1 - \lambda_0) \sigma^2 / 2 & |r_i| > \lambda_2 \sigma \end{cases} \tag{3}$$

$r_i = y_i - X_i^{\tau}\hat{\theta}$
$\lambda_0 = 3,\ \lambda_1 = 4,\ \lambda_2 = 6.$

Equation (2) can be rewriten as:

$$\sum_{i=1}^{N} X_i \Psi\{(y_i - X_i^{\tau}\hat{\theta})/\sigma\} = 0 \tag{4}$$

where

$$\Psi\{\cdot\} = F'\{\cdot\}.$$

Then

$$\hat{\theta} = \left[\sum_{i=1}^{N} X_i W_i X_i^{\tau}\right]^{-1} \left[\sum_{i=1}^{N} X_i W_i y_i\right] \tag{5}$$

and

$$W_i = \Psi(r_i/\sigma)/(r_i/\sigma). \tag{6}$$

From (3) we have

$$W(r_i/\sigma)=\begin{cases}1 & |r_i|\le\lambda_0\sigma \\ \lambda_0\sigma/|r_i| & \lambda_0\sigma<|r_i|\le\lambda_1\sigma \\ \lambda_0(\lambda_2\sigma-|r_i|)/(\lambda_2-\lambda_1)|r_i| & \lambda_1\sigma<|r_i|\le\lambda_2\sigma \\ 0 & |r_i|>\lambda_2\sigma\end{cases} \quad (7)$$

and

$$\hat{\sigma}^2=\frac{1}{N-p}\sum_{i=1}^{N} r_i^2 \quad (8)$$

p is number of the paramaters estimated.

When given the starting values θ_0 and σ_0 ,we can solve $\hat{\theta}$ by (5),(6) and (7). The solution is then iterated between (5) and (8) until convergence of the object function.

$$|U^{j+1}-U^j| \;/\; U^j < 10^{-3}$$

here

$$U=\sum_{i=1}^{N} F\{(y_i-X_i^T\hat{\theta})/\sigma\}$$

j is the times of the iteration.

PROCEDURES

The predicted and observed ranges is R_c and R_o at each instant of observation T. After atmospheric correction, center of mass correction and delay calibration, we have the following range residual equation:

$$\begin{aligned} y_i &= \Delta R_i \\ &= a+b\dot{\rho}_i + e_i. \end{aligned} \quad (9)$$

Where a, b are range bias and time bias.

$\dot{\rho}$ is the variability of range.

Reference show a mathod of caculation which have 50% breakdown point.

a) If totel observation data points are N ,which are divided into n subgroups equaly and every subgroup inclodes four data points, as:

y_1	y_{n+1}	y_{2n+1}	y_{3n+1}
y_2	y_{n+2}	y_{2n+2}	y_{3n+2}
......			

y_k $\quad y_{n+k} \quad y_{2n+k} \quad y_{3n+k}$

......

y_n $\quad y_{n+n} \quad y_{2n+n} \quad y_{3n+n}$

$n=N/4.$

When noise numbers in raw obsrevation data are less then N/2, there must be a subgroup in which contains one noise point at most.

b) For the linear model as (9), we can find the linear estimated value of $\hat{b}_k$ for any subgroup k:

$$\hat{b}_k = \sum_{l=1}^{4} \beta_l y_l \tag{10}$$

If $\hat{b}_k$ is no-bias, we have:

$$\sum_{l=1}^{4} \beta_l = 0$$

$$\sum_{l=1}^{4} \beta_l \dot{\rho} = 1 \tag{11}$$

and

$$\sum_{l=1}^{4} \frac{1}{\delta_l} \beta_l^2 = \min$$

where

$$\delta_2 = \delta_3 = 1$$

$$\delta_1 = \frac{T_2}{2T_1 + T_2} C \qquad \text{(C is a constant to be selected)}$$

$$\delta_4 = \frac{T_2}{2T_3 + T_2} C$$

$$T_l = \dot{\rho}_{l+1} - \dot{\rho}_l \qquad (l=1, 2, 3)$$

By solving equations (11), we get:

$$\beta_l = \delta_l \lambda \tau_l \qquad (l=1, 2, 3, 4)$$

where

$$\lambda = 1 / \sum_{l=1}^{4} \delta_l \tau_l^2$$

$$\tau_l = \dot{\rho}_l - \dot{\rho}_0$$

$$\dot{\rho}_0 = \sum_{l=1}^{4} \delta_l \dot{\rho}_l / \sum_{l=1}^{4} \delta_l$$

$$C = \max\left[\frac{T_2 + T_3}{T_1 + T_2 + T_3}, \frac{T_1 + T_2}{T_1 + T_2 + T_3}\right]$$

Thus, the residuats of k-subgroup are

$$r_{kl}=y_{kl}-\hat{b}_k\dot{\rho}_{kl} \qquad (l=1,2,3,4)$$

c) For each subgroup, the largest and smallest values of r_{kl} are rejeated. And we can get the initial values a_{ko}, b_{ko} from remained two data points through follows:

$$y_{kl}=a_{ko}+b_{ko}\dot{\rho}_{kj} \qquad (j=1,2)$$

d) Then calcuiation the object function of M-estimate used all observations for every subgroups:

$$U_k=\sum_{i=1}^{N}F\{y_i-a_{ko}-b_{ko}\dot{\rho}_i\} \qquad (k=1,2,\ldots n)$$

where $F\{\cdot\}$ can be taken from (3), and the initial value of σ can be arbitrarily given, for example 0.5 meters.

e) Select the minimum value from U_k $(k=1,2,\ldots n)$. Suppose k=m, that is

$$U_m=\min.$$

Then a_{m0} and b_{m0}, those are taken from m-subgrop, can be used as the initial values a_0, b_0. It is sure that the a_0 and b_0 are taken from 'good' observation points.

f) Then we can get

$$r_i=y_i-a_0-b_0\dot{\rho}_i \qquad (i=1,2,\ldots N)$$

$$\sigma_0^2=\frac{1}{N-2}\sum_{i=1}^{N}r_i.$$

Because a_0, b_0 are obtaied by two data points, they have just lower accuracy. From (5) to (8) and iterated until covergence, the accurate results a, b can be get as above.

g) After correcting range bias and time bias, we can get a polynomial in time as following:

$$\Delta\rho_i'=y_i-a-b\dot{\rho}_i$$
$$=a_0+a_1t_i+a_2t_i^2+a_3t_i^3+\ldots\ldots$$

Useing M-estimator, the parameters of polynomial $a_0, a_1, a_2, a_3\ldots\ldots$ can be solved.

CONCLUSION AND DISCUSSION

Comparing with the LS estimator, M-estimator has its advantage as follows:

a) It can be preprocess observation data that contain a large amount of noises, for example , a pass for LAGEOS in daytime are shown in fig 1, (12/20/1991 8:45 UT). In this pass rate of noise is up to 70%.

b) At same accuracy, the order of polynomial fitting is only 4 using M-estimator, while the order is up to 6-8 or more with LS estimator. Seeing table 1.

c) Noise mixed at the parts near the both ends of the curve can be detected and deleted.

Besides, comparison with the method of screen-processin and LS estimator, one third time is saved with M-estimator.

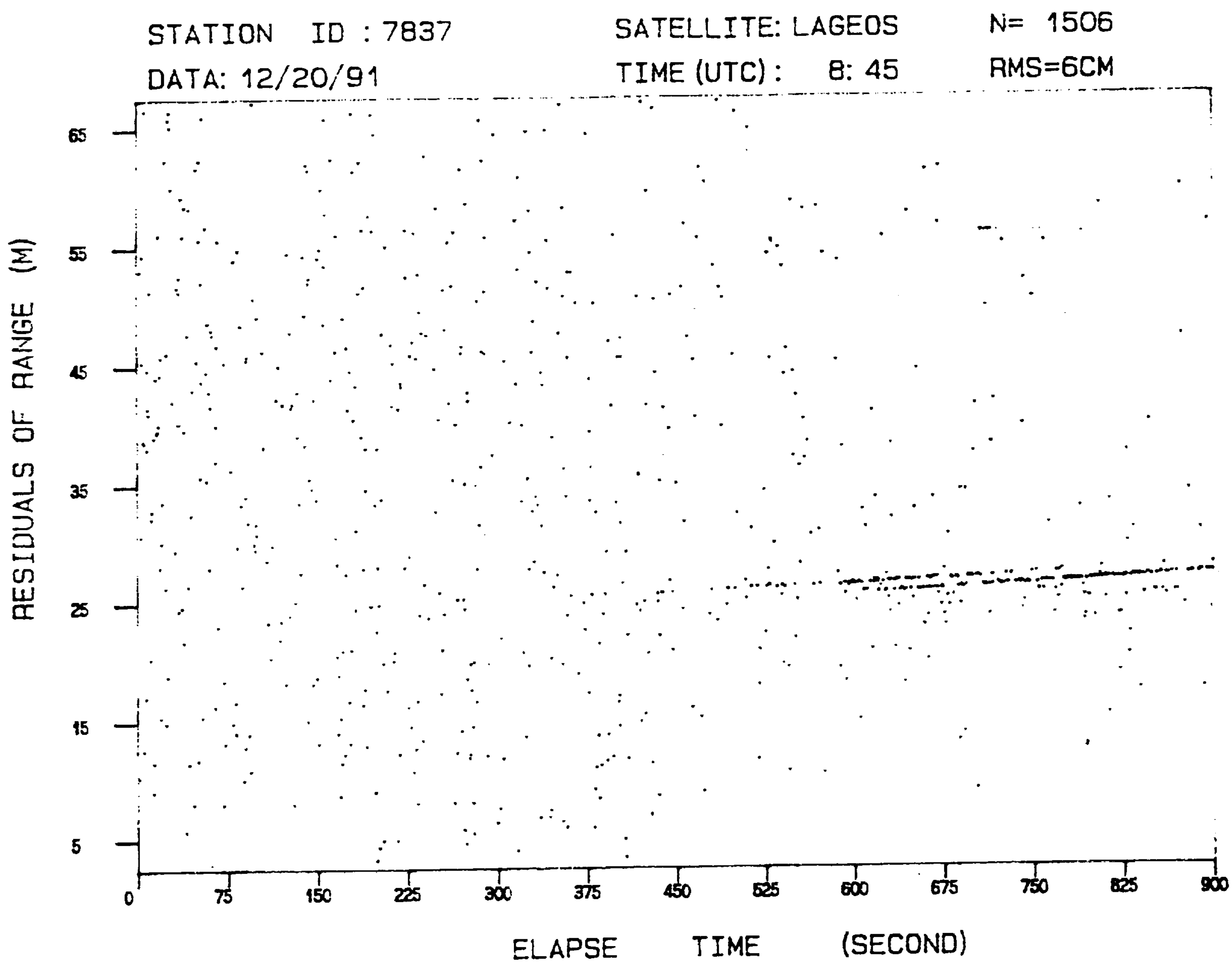
REFERENCE

J. peizhang On Joint Robust Estimation of The Parameters and The Variance. Acta Astronomica Sinica. Vol. 33. No 1. 1992.

Table 1.
Comparison for Two Estimators(Lageos)

Passes	M-estimator			LS-estimator		
	Order	RMS(cm)	Points	Order	RMS(cm)	Points
Y M D H						
92011011	4	5.8	28	8	5.9	29
92011110	4	5.9	778	8	5.7	752
92011120	4	5.1	452	8	5.4	419
92011317	4	6.0	216	4	6.6	212
92011321	4	4.8	170	4	5.0	169
92011416	4	4.6	169	8	5.1	169
92011419	4	5.9	94	8	5.9	86
92011512	4	5.4	457	8	5.8	453
92022216	4	6.2	187	8	6.1	183
92031116	4	5.6	326	8	6.1	326
92041514	4	5.3	425	8	5.9	422
92041616	4	4.9	419	8	7.0	417
92042011	6	5.6	60	8	5.3	56
92042018	4	5.3	41	4	5.8	41
92042613	4	5.1	583	4	6.4	585
92043015	4	4.1	77	8	5.3	83
92050815	4	3.1	91	8	2.9	91
92052114	4	2.7	212	16	2.7	210
92052312	4	2.6	170	12	2.5	170
92060216	4	3.2	503	8	2.9	471

Fig. 1.
Residual for A Lageos Pass in Daytime



Lunar Laser Ranging